

L28 The inverse Trigonometric functions (Conti.) (續.反三角函數)

Ch.8 Techniques of integrations

8.1 Review (配方法)

8.2 Integration by parts (分部積分)

Let  $y = \sec^{-1} x$ , then  $\sec y = x$ ,  $y \in [0, \pi/2) \cup (\pi/2, \pi]$ .

$$(\sec^{-1} x)' = 1/\sec y \tan y = 1/\sec y \sqrt{\sec^2 y - 1} = 1/|x| \sqrt{x^2 - 1}$$

$\therefore y \in (0, \pi/2) \cup (\pi/2, \pi) \therefore \sin y > 0 \Rightarrow \sec y \tan y > 0$

$$\sec y = 1/\cos y, \tan y = \sin y/\cos y$$

$$\text{Thm: } (\sec^{-1} x)' = \frac{1}{|x| \sqrt{x^2 - 1}} \text{ on } (-\infty, -1) \cup (1, \infty)$$

$$\text{Thm: } \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

pf:

型式上不太一樣，這個東西怎麼證。我們在算不定積分有兩種方式，如果你告訴我答案，有一種方式是 antiderivative 的定義，也就是用不定積分的定義來寫。證(B)是(A)的 antiderivative。牽涉到絕對值得微分，要分開討論。

$$(1) \text{ for } x > 0, \left( \frac{1}{a} \sec^{-1} \frac{|x|}{a} \right)' = \left( \frac{1}{a} \sec^{-1} \frac{x}{a} \right)' = \frac{1}{a} \frac{1}{\frac{x}{a} \sqrt{\frac{x^2}{a^2} - 1}} \cdot \frac{1}{a} = \frac{1}{x \sqrt{x^2 - a^2}}$$

(2) for  $x < 0$ ,

eg.

$$\textcircled{1} [\sec^{-1}(2 \ln x)]' = \frac{1}{2 |\ln x| \sqrt{4 \ln^2 x - 1}} \cdot \frac{2}{x} =$$

$$\textcircled{2} \int_{2\sqrt{2}}^4 \frac{dx}{x \sqrt{x^2 - 4}} = \frac{1}{2} \sec^{-1} \frac{|x|}{2} \Big|_{2\sqrt{2}}^4 = \frac{1}{2} \sec^{-1} 2 - \frac{1}{2} \sec^{-1} \sqrt{2} = \frac{\pi}{6} - \frac{\pi}{8}$$

(VI)  $\sec^{-1}: (-\infty, 1] \cup [1, \infty) \rightarrow (-\pi/2, 0) \cup (0, \pi/2)$

$$\text{Thm: } (\sec^{-1})' = -\frac{1}{|x| \sqrt{x^2 - 1}} \text{ on } (-\infty, -1) \cup (1, \infty)$$

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Ex:P358(6.8.18.24.25.35.36.45.47.48.49.50.53.57.61)

Chapter8 Thchniques of integrations

§ 8.1 Review

eg.  $\int \frac{dx}{x^2 + 2x + 5} =$

$$\int \frac{dx}{x^2 + 2x + 5} = \int \frac{dx}{(x+1)^2 + 4} = \frac{1}{4} \int \frac{dx}{\left(\frac{x+1}{2}\right)^2 + 1} = \frac{1}{2} \tan^{-1} \frac{x+1}{2} + C$$

Let  $u=x+1/2$ , then  $du=1/2dx$

Ex:補充題:

①  $\int \frac{dx}{\sqrt{3-4x^2}}$    ②  $\int \frac{dx}{\sqrt{e^{2x}-6}}$    ③  $\int \frac{dx}{\sqrt{4x-x^2}}$    ④  $\int \frac{dx}{4x^2+4x+2}$

§ 8.2 Integration by parts

$$\therefore f(x)g'(x)+f'(x)g(x)=(f(x)g(x))'$$

$$\therefore \int_a^b f(x)g'(x)dx + \int_a^b f'(x)g(x)dx = \int_a^b [f(x)g(x)]' dx$$

經過移項而得到的結果就是 Integration by parts

Q:  $\int_a^b [f(x)g(x)]' dx$  為什麼可積，用哪一個定理？

A: The second Fundamental thm. of Integral calculus。找 antiderivative

Thm: Integration by parts

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)dx$$

Thm:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

把一個不好積的型式換成一個比較好積的，中間需要調整。(先積後微)。

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口語：先積後微分，一次只做一項，減號銜接，放入積好的東西

什麼時候用 Integrations by parts ? 先看 substitution 積分積不出來。

eg.

$$\textcircled{1} \int x e^x dx = u=x, dv=e^x$$

$$= x e^x - \int 1 \cdot e^x = x e^x - e^x + C = e^x (x-1) + C$$

$$\textcircled{2} \int x \sin(2x) dx = u=x, dv=\sin(2x)$$

$$= \frac{1}{2} \int x 2 \sin(2x) dx = \frac{1}{2} [-x \cos(2x) + \int \cos(2x) dx] = -\frac{1}{2} x \cos(2x) + \frac{1}{4} \sin(2x) + C$$

$$\textcircled{3} \int_1^2 x^3 \ln x dx = u=\ln x, dv=x^3$$

$$= \int_1^2 x^3 \ln x dx = \frac{1}{4} x^4 \ln x \Big|_1^2 - \frac{1}{4} \int_1^2 x^4 \cdot \frac{1}{x} dx$$

$$\textcircled{4} \int \ln x dx = u=\ln x, dv=dx$$

$$= x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$$

$$\textcircled{5} \int x^5 e^{x^3} dx = u=x^3, dv=3x^2 e^{x^3}$$

$$= \frac{1}{3} \int x^3 (3x^2 e^{x^3}) dx = \frac{1}{3} [x^3 e^{x^3} - \int 3x^2 e^{x^3} dx] = \frac{1}{3} x^3 e^{x^3} - e^{x^3} + C$$

$$\textcircled{6} \int e^x \cos x dx = u=\cos x, dv=e^x$$

$$= e^x \cos x + \int \sin x e^x dx = e^x \cos x + e^x \sin x - \int e^x \cos x dx$$

$$\Rightarrow \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$$

$$\textcircled{7} \int \sin^{-1} x dx = u=\sin^{-1} x, dv=dx$$

$$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx = x \sin^{-1} x + \frac{1}{2} \cdot 2\sqrt{1-x^2} + C$$

Ex.P408(4.7.21.29.32.33.38.40.45.68.74.76.77.78)